Problem 6-16

Location of the compsite centroid:

$$\bar{z} = \frac{1}{m_0 + m}\left\{ m\left(\frac{3}{8}R + \frac{h}{4}\right)\right\}$$

where the centroid is measured upward from the centroid of the hemisphere.

From table 6-3 the centroidal inertia tensors for the cone and hemisphere are:

$$[I]_{C_1}^{cone} = \begin{bmatrix} \frac{3}{80}m\left(4R^2 + h^2\right) & 0 & 0 \\ 0 & \frac{3}{80}m\left(4R^2 + h^2\right) & 0 \\ 0 & 0 & \frac{3}{10}mR^2 \end{bmatrix}$$

$$[I]_{C_2}^{hemisphere} = \begin{bmatrix} 0.259m_0R^2 & 0 & 0 \\ 0 & 0.259m_0R^2 & 0 \\ 0 & 0 & \frac{2}{5}m_0R^2 \end{bmatrix}$$

using the parallel axis theorem, the inertia tensor for the composite body is:

$$[I]_C = [I]_{C_1}^{cone} + m\begin{bmatrix} \left(\frac{3}{8}R + \frac{1}{4}h - \bar{z}\right)^2 & 0 & 0 \\ 0 & \left(\frac{3}{8}R + \frac{1}{4}h - \bar{z}\right)^2 & 0 \\ 0 & 0 & 0 \end{bmatrix} + [I]_{C_2}^{hemisphere} + m_0\begin{bmatrix} \bar{z}^2 & 0 & 0 \\ 0 & \bar{z}^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$