Proceedings of the American Society of Composites, Seventeenth Technical Conference, October 21-23, 2002, P

Modeling and Analysis of Tapered Sandwich Beams

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ABSTRACT

Structural sandwich construction is used in many air and space vehicles, cargo containers, boats and ships. Connection of the sandwich construction component to a framework or substructure is a critical issue in the detail design for sandwich construction. The tapered connection where the facings are drawn together at the support is one of the most efficient types of connections in sandwich construction. Analytical models have shown that the behavior of the tapered section is counterintuitive and that, for a tapered cantilever sandwich beam with fixed dimensions at the clamped edge, there is an optimum taper angle where the tip deflection is a minimum. This optimal angle is heavily dependent upon the relative stiffness of the facing and core materials. This decrease in deformation with increasing taper angle is due to the participation of the facings in resisting transverse shear loads. We present a tapered sandwich theory that is simple to use, yet accurately predicts the stresses and deflection of both symmetric and non-symmetric tapered sections. The stiffness matrix for a tapered sandwich member exhibits bending-shear and extension-shear elastic coupling. Unlike a sandwich structure of constant depth, an applied bending moment and/or axial load will cause shear deformation of the core in a tapered sandwich member. The shear stress at the interface between the core and facings is computed by integrating the three-dimensional equilibrium equations in a direction perpendicular to the facing surfaces. Results from the tapered sandwich theory show very good comparison with finite element models for several case studies.

Keywords: Sandwich panels; tapered joint; tapered sandwich beam; honeycomb core.

INTRODUCTION

Sandwich construction is one of the most functional forms of composite structures developed by the composite industry. It has attained broad acceptance in aerospace and many other industries and it is widely employed in aircraft and space vehicles, ships, boats, cargo containers and residential construction. Sandwich construction provide several key benefits over conventional structures, such as very high bending stiffness, low weight, cost effectiveness and durability. The major advantage of this structural type is a very high stiffness-to-weight ratio and high bending strength. A typical sandwich beam or panel usually consists of honeycomb, foam or low-density wood cores sandwiched between isotropic or laminated facings. When laminated facings are used, they may be

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designed to have quasi-isotropic, orthotropic or anisotropic properties. The facings are designed to carry primarily the bending stresses while the core resists the shear loads and increases the flexural stiffness of the structure by holding the facings apart. The structural analysis of constant-thickness sandwich composite structures is discussed in books by Whitney [1] and Vinson [2], where they emphasize the importance of including the shear flexibility of the core. In some applications, such as in the design of aerospace vehicles, it is necessary to use variable-thickness sandwich construction, either locally or globally, for functional and/or aerodynamic reasons. The tapered sandwich connection, where the facings are drawn together at the support to connect the sandwich component to a framework is another reason for using a variable-thickness construction. Use of a tapered connection in sandwich structural components typically leads to a substantial reduction in construction depth. This type of connection was studied experimentally and analytically by Caccese and Gauthier [3, 4] for its potential use in the aeroshell structure of the NASA X-38. Kuczma and Vizzini [5] have investigated the failure modes and load distributions in tapered sandwich-to-laminate specimens under tensile, compressive and bending loads and the experimental data were correlated with three-dimensional finite element models. In general, experiments have shown that initial damage in tapered sandwich connections occurs at the root of the taper resulting in delamination of the facings from the core. When dealing with homogeneous beams of variable thickness, it is usually assumed that the constant-thickness moment-curvature relationships of beam theory are still valid, provided we use the bending rigidity based on the local thickness. Huang and Alspaugh [6] used a constant-thickness sandwich theory, with stiffnesses that varied in accordance with the local thickness, to study sandwich beams of variable thickness. It was shown by Libove and Lu [7, 8] that this approach can lead to significant errors since the membrane stresses in the facings have a transverse shear component which alters the transverse shear load in the core and hence the transverse shear deformation. The theory was extended by Paydar and Libove [9, 10] to study the general bending of isotropic sandwich plates of variable thickness and by Lu [11] to analyze the tapered sandwich beams consisting of fiber-reinforced anisotropic facings and honeycomb core. Peled and Frostig [12] have rigorously developed a theory for the analysis of tapered sandwich beams with transversely flexible core. Their analysis accounts for higher-order effects in the form of nonlinear displacements fields through the thickness of the sandwich beam which are pronounced in the vicinity of concentrated loads or supports as well as at the ends of tapered transition zones.

In order to design and use tapered sandwich composite construction in practical applications, it is essential to accurately compute the stresses and deflections. Reliable estimates of the stresses, in conjunction with a good failure theory, are needed to predict the maximum load carrying capacity of a tapered sandwich structure. The objective of the present investigation is to develop a simple tapered sandwich theory in which the reference surface strains and curvatures are related to the applied loads through the familiar A, B and D matrices. Unlike sandwich beams of uniform thickness, tapered sandwich beams exhibit bending-shear and extension-shear elastic couplings. The proposed theory accurately predicts the stresses and deflection of both symmetric and non-symmetric tapered sections. In this investigation, we consider tapered sandwich structures consisting of isotropic facings and honeycomb core. Results from the tapered sandwich theory show very good comparison with plane strain finite element models for several case studies.

PROBLEM FORMULATION

The analytical development of tapered composite members starts with the general depiction of a tapered sandwich beam as shown in Figure 1. A rectangular Cartesian coordinate system, denoted by x-y-z in Fig. 1a, is used to describe the deformation of a tapered sandwich beam with isotropic facings. The thickness is assumed to vary linearly along the span (x-direction). It is assumed that the facings are relatively thin compared to the core and therefore behave as membranes. The core is assumed to be inextensible in the thickness direction and to carry only transverse shear stresses. Since the facings are tapered, it is advantageous to use a local coordinate system, denoted by s-y-n in Fig. 1a, such that the s- and n-axes are parallel and normal to the facing, respectively. The taper angle of the facing, ϕ, is taken as positive counter-clockwise with respect to the x-axis. The tapered structure is composed of three distinct layers consisting of the core and two facings. The layers are numbered from bottom to top as shown in Fig. 1b. The z-coordinate of the bottom surface of the kth layer is designated z_k with the top surface of the layer being z_{k+1}.

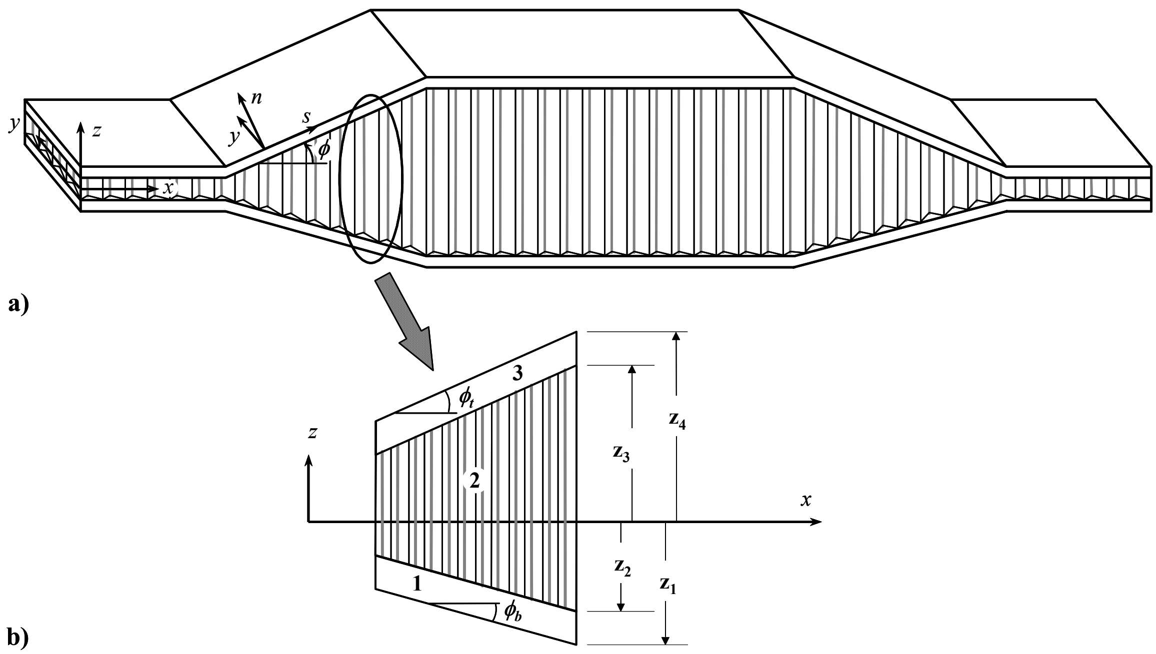

Figure 1 Coordinate system and layer numbering for a tapered sandwich structure.

In general, the z_k's are functions of the x-coordinate since the sandwich beam is tapered. The taper angle of the top and bottom facings are $\phi = \phi_t$ and $\phi = \phi_b$, respectively.

ANALYTICAL MODEL

Since the facings are relatively thin compared to the core, we assume that the facings are in a state of plane stress. This is a reasonable assumption since the stress components in the plane of the facing are generally much larger than the stress components perpendicular to the plane. For tapered sandwich structures, the plane stress conditions are

$$\sigma_{nn} = \tau_{ns} = \tau_{ny} = 0, \tag{1}$$

where the normal stress components are denoted by σ and the shear stress components by τ. The stress-strain relations for the isotropic facings in the (s, y, n) coordinate system are

$$\begin{bmatrix} \sigma_{ss} \\ \sigma_{yy} \\ \sigma_{nn} \\ \tau_{ny} \\ \tau_{ns} \\ \tau_{sy} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_{ss} \\ \varepsilon_{yy} \\ \varepsilon_{nn} \\ \gamma_{ny} \\ \gamma_{ns} \\ \gamma_{sy} \end{bmatrix}, \tag{2}$$

where C_{ij} are the elastic stiffness components. The elastic stiffness components for the isotropic facings in terms of the Young's moduli E, Poisson's ratio ν and shear moduli G are $C_{11} = C_{22} = C_{33} = E(1-\nu)/(1+\nu)(1-2\nu)$, $C_{12} = C_{13} = C_{23} = E\nu/(1+\nu)(1-2\nu)$ and $C_{44} = C_{55} = C_{66} = G = E/2(1+\nu)$. The stresses and strains in the (x, y, z) and (s, y, n) coordinate systems are related as

$$\{\sigma\}_{syn} = [R^+]\{\sigma\}_{xyz}, \quad \{\sigma\}_{xyz} = [R^-]\{\sigma\}_{syn}, \tag{3}$$

$$\{\varepsilon\}_{syn} = [R^-]^T\{\varepsilon\}_{xyz}, \quad \{\varepsilon\}_{xyz} = [R^+]^T\{\varepsilon\}_{syn}, \tag{4}$$

where $\{\sigma\}_{xyz} = \{\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \tau_{yz}, \tau_{xz}, \tau_{xy}\}^T$, $\{\varepsilon\}_{xyz} = \{\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{zz}, \gamma_{yz}, \gamma_{xz}, \gamma_{xy}\}^T$, $\{\sigma\}_{syn} = \{\sigma_{ss}, \sigma_{yy}, \sigma_{nn}, \tau_{ny}, \tau_{ns}, \tau_{sy}\}^T$ and $\{\varepsilon\}_{syn} = \{\varepsilon_{ss}, \varepsilon_{yy}, \varepsilon_{nn}, \gamma_{ny}, \gamma_{ns}, \gamma_{sy}\}^T$ are column vectors of stresses and strains in the (x, y, z) and (s, y, n) coordinate systems, respectively. Matrices $[R^+]$ and $[R^-]$ are the transformation matrices defined by

$$[R^\pm] = \begin{bmatrix} \cos^2\phi & 0 & \sin^2\phi & 0 & \pm 2\cos\phi\sin\phi & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ \sin^2\phi & 0 & \cos^2\phi & 0 & \mp 2\cos\phi\sin\phi & 0 \\ 0 & 0 & 0 & \cos\phi & 0 & \mp\sin\phi \\ \mp\cos\phi\sin\phi & 0 & \pm\cos\phi\sin\phi & 0 & \cos^2\phi - \sin^2\phi & 0 \\ 0 & 0 & 0 & \pm\sin\phi & 0 & \cos\phi \end{bmatrix}, \tag{5}$$

where ϕ is the taper angle of the facing. That is, $\phi = \phi_t$ for the top facing and $\phi = \phi_b$ for the bottom facing.

As in the case of uniform sandwich beams of constant depth (see Whitney [1], Vinson [2]), an analytical model for tapered beams is developed by assuming that the strains in the facings are linear functions of the thickness coordinate:

$$\begin{aligned} \varepsilon_{xx} &= \varepsilon^0_{xx}(x) + z\ \kappa^0_{xx}(x), \\ \varepsilon_{yy} &= \varepsilon^0_{yy}(x) + z\ \kappa^0_{yy}(x), \\ \gamma_{xy} &= \gamma^0_{xy}(x) + z\ \kappa^0_{xy}(x). \end{aligned} \tag{6}$$

The transverse normal strain ε_{zz}, and the transverse shear strains γ_{xz} and γ_{yz} in the facings are obtained from ε_{xx}, ε_{yy} and γ_{xy} by recognizing that the facings are in a state of plane stress. The stresses in the (s, y, n) coordinate system are obtained through $\{\sigma\}_{syn} = [C]\{\varepsilon\}_{syn} = [C][R^-]^T\{\varepsilon\}_{xyz}$. The three plane stress equations (1) are enforced to obtain ε_{zz}, γ_{xz} and γ_{yz} in terms of ε_{xx}, ε_{yy} and γ_{xy} as

$$\begin{aligned} \varepsilon_{zz} &= \frac{(1 - 2\nu - \cos 2\phi)\varepsilon_{xx} - 2\nu\varepsilon_{yy}\cos 2\phi}{1 - 2\nu + \cos 2\phi}, \\ \gamma_{xz} &= \frac{2(\varepsilon_{xx} + \nu\varepsilon_{yy})\sin 2\phi}{1 - 2\nu + \cos 2\phi}, \quad \gamma_{yz} = \gamma_{xy}\tan\phi. \end{aligned} \tag{7}$$

The stress-strain relations in the (x, y, z) coordinate system for the tapered facing, computed using (7), $(4)_1$, (2), $(3)_2$, are

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \tilde{Q}_{11} & \tilde{Q}_{12} & 0 \\ \tilde{Q}_{21} & \tilde{Q}_{22} & 0 \\ \tilde{Q}_{31} & \tilde{Q}_{32} & 0 \\ 0 & 0 & \tilde{Q}_{46} \\ \tilde{Q}_{51} & \tilde{Q}_{52} & 0 \\ 0 & 0 & \tilde{Q}_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix}, \tag{8}$$

where $\tilde{Q}_{ij}$ are the *plane stress reduced stiffnesses for tapered isotropic facings*, defined by

$$\begin{aligned} &\tilde{Q}_{11} = 2E\cos^2\phi/D,\ \tilde{Q}_{12} = 2E\nu\cos^2\phi/D,\ \tilde{Q}_{21} = 2E\nu/D, \\ &\tilde{Q}_{22} = E + 2E\nu^2/D,\ \tilde{Q}_{31} = 2E\sin^2\phi/D,\ \tilde{Q}_{32} = 2E\nu\sin^2\phi/D, \\ &\tilde{Q}_{46} = G\tan\phi,\ \tilde{Q}_{51} = E\sin 2\phi/D,\ \tilde{Q}_{52} = E\nu\sin 2\phi/D,\ \tilde{Q}_{66} = G, \end{aligned} \tag{9}$$

where $D = (1+\nu)(1-2\nu+\cos 2\phi)$.

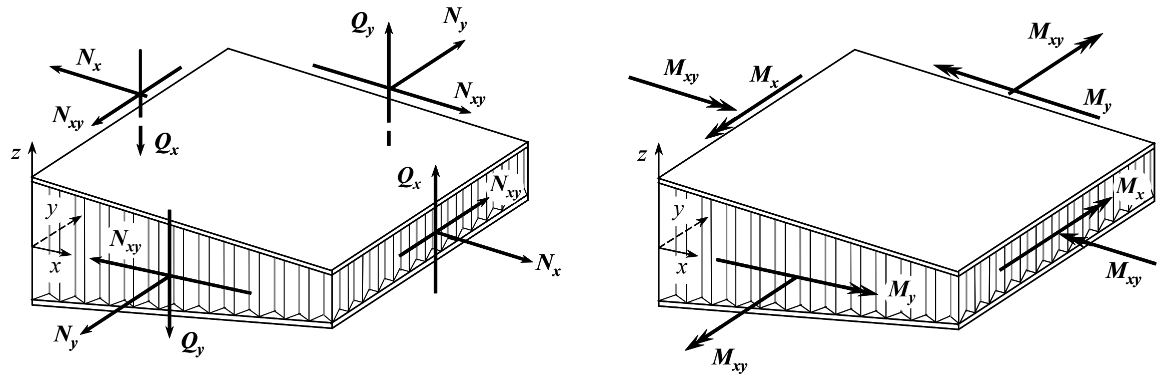

Figure 2 Force and moment resultants for a tapered sandwich element.

The core is made of an orthotropic material and its primary function is to space and stabilize the facings and transfers shear between them. The stresses σ_{xx}, σ_{yy} and τ_{xy} of the core are assumed to be negligible compared to that in the facings. The transverse shear stresses τ_{xz} and τ_{yz} in the core are assumed to be constant throughout the thickness and they are related to the core shear strains by

$$\tau_{xz}^{(2)} = G_{xz}^c \gamma_{xz}^c,\ \tau_{yz}^{(2)} = G_{yz}^c \gamma_{yz}^c, \tag{10}$$

where G_{xz}^c and G_{yz}^c are the transverse shear moduli, and γ_{xz}^c and γ_{yz}^c are the transverse shear strains of the core. The stress resultants are defined as:

$$\begin{aligned} &[N_x, N_y, N_{xy}] = \int_{z_1(x)}^{z_4(x)} [\sigma_{xx}, \sigma_{yy}, \tau_{xy}] dz, \quad [Q_x, Q_y] = \int_{z_1(x)}^{z_4(x)} [\tau_{xz}, \tau_{yz}] dz, \\ &[M_x, M_y, M_{xy}] = \int_{z_1(x)}^{z_4(x)} z[\sigma_{xx}, \sigma_{yy}, \tau_{xy}] dz, \end{aligned} \tag{11}$$

where the quantities N_x, N_y and N_{xy} are the in-plane force resultants, Q_x and Q_y are the transverse shear load resultants and M_x, M_y and M_{xy} are the moment resultants, as depicted in Fig. 2.

Substitution of the facing stresses from (8) and the core stresses from (10) into (11) leads to the following matrix equation for the resultant forces and moments in terms of the reference surface strains and curvatures

$$\begin{Bmatrix} N_x \\ N_y \\ Q_y \\ Q_x \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & 0 & 0 & B_{11} & B_{12} & 0 \\ A_{21} & A_{22} & 0 & 0 & 0 & B_{21} & B_{22} & 0 \\ 0 & 0 & A_{44} & 0 & A_{46} & 0 & 0 & B_{46} \\ A_{51} & A_{52} & 0 & A_{55} & 0 & B_{51} & B_{52} & 0 \\ 0 & 0 & 0 & 0 & A_{66} & 0 & 0 & B_{66} \\ B_{11} & B_{12} & 0 & 0 & 0 & D_{11} & D_{12} & 0 \\ B_{21} & B_{22} & 0 & 0 & 0 & D_{21} & D_{22} & 0 \\ 0 & 0 & 0 & 0 & B_{66} & 0 & 0 & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^0 \\ \varepsilon_{yy}^0 \\ \gamma_{yz}^c \\ \gamma_{xz}^c \\ \gamma_{xy}^0 \\ \kappa_{xx}^0 \\ \kappa_{yy}^0 \\ \kappa_{xy}^0 \end{Bmatrix}, \tag{12}$$

where A_{ij} are the extensional and shear rigidities, D_{ij} are the bending rigidities and B_{ij} are the bending-extension rigidities which are defined as follows,

$$\begin{aligned} &[A_{ij}(x), B_{ij}(x), D_{ij}(x)] = \sum_{k=1,3} \int_{z_k(x)}^{z_{k+1}(x)} \tilde{Q}_{ij}^{(k)} [1, z, z^2] dz; \quad i,j = 1,2,6, \\ &[A_{44}(x), A_{55}(x)] = [G_{yz}^c, G_{xz}^c](z_3(x) - z_2(x)). \end{aligned} \tag{13}$$

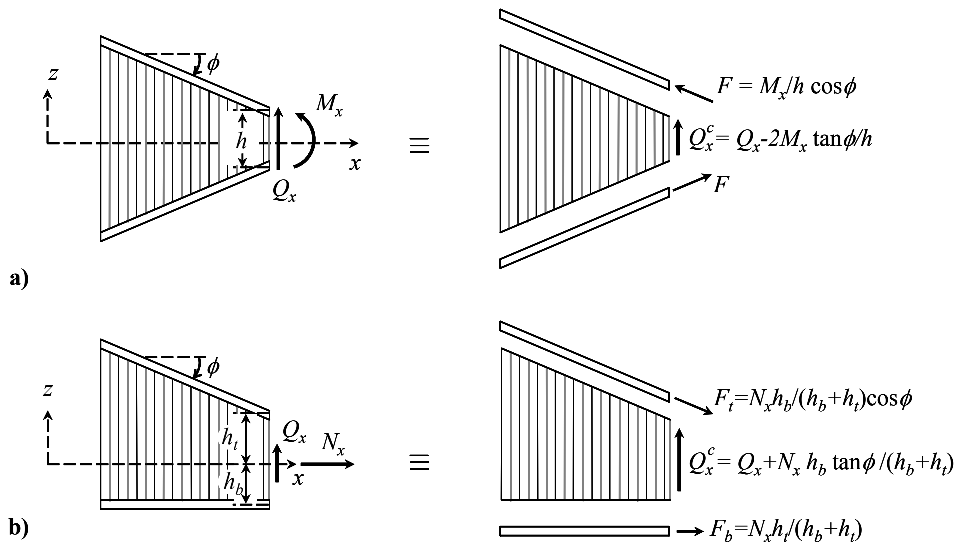

Figure 3 Elastic couplings in tapered sandwich beams: a) bending-shear coupling and b) extension-shear coupling.

There are two cases of laminated plates that can be treated as one-dimensional problems: (1) tapered sandwich beams, and (2) cylindrical bending of tapered members. When the width of the beam along the y-axis is smaller than the length along the x-axis, it is treated as a sandwich beam. For a sandwich beam in bending, the transverse shear load Q_x and bending moment M_x are assumed to be known, whereas the other loads vanish, i.e., $N_x = N_y = Q_y = N_{xy} = M_y = M_{xy} = 0$. The reference surface strains, curvatures and transverse shear strains can be computed from Eq. (12) and the stresses are computed using Eq. (8). In cylindrical bending, the laminated plate is assumed to be a plate strip that is very long along the y-axis and has a finite dimension along the x-axis. All derivatives with respect to y are zero and the mid-surface of the tapered member bends into a cylindrical shape. For cylindrical bending, the transverse shear load Q_x and bending moment M_x are assumed to be known, $N_x = Q_y = N_{xy} = M_{xy} = 0$, $\varepsilon_{yy}^0 = \kappa_{yy}^0 = 0$ and ε_{xx}^0, N_y, γ_{yz}^c, γ_{xz}^c, γ_{xy}^0, κ_{xx}^0, M_y and κ_{xy}^0 are determined from Eq. (12). The deflection of the sandwich member is computed using the energy method. The transverse shear stress τ_{ns} in the facings is computed by integrating the three-dimensional equilibrium equation along a path that is perpendicular to the facings:

$$\tau_{ns} = -\int (\sigma_{ss,s} + \tau_{sy,y})\, dn. \tag{14}$$

The integration constant in (14) is determined by recognizing that τ_{ns} vanishes on the top surface $z = z_4(x)$ and bottom surface $z = z_1(x)$ of the sandwich member.

BENDING-SHEAR AND EXTENSION-SHEAR ELASTIC COUPLINGS

An important feature of tapered sandwich members is the elastic coupling between transverse shear and bending due to elastic coupling stiffness B_{51} in Eq. (12), which vanishes for sandwich beams with constant depth. The physical meaning of the shear-bending elastic coupling can be understood through a simple analysis of the loads acting on a cross-section of a symmetric tapered member in Fig. 3a. Consider the case when the tapered sandwich member is subjected to an internal moment M_x and shear force Q_x. The assumption is made that the normal force due to the bending moment is transmitted through the facings only. A portion of the resultant shear force is transmitted through the facings due to their angle of inclination. The core resists the reminder of the shear force. As can be seen, even when the shear load Q_x is absent, a shear force Q_x^c is induced in the core due to the inclination of the facings, thus coupling the bending and

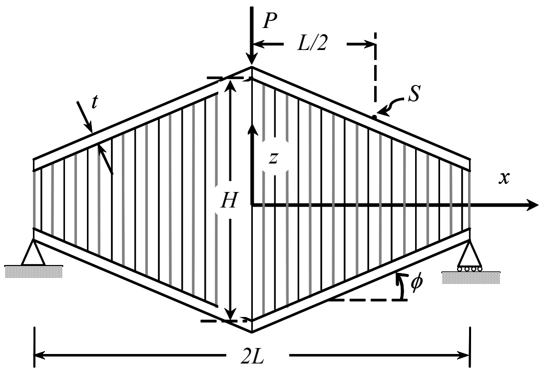

Figure 4 Geometry of a symmetric tapered sandwich member.

shear modes. If the tapered sandwich beam is unsymmetric, as depicted in Fig. 3b, an axial force would cause a transverse shear load in the core due to the angle of inclination of the facings. The elastic rigidity A_{51} quantifies the extension-shear coupling effect.

NEGATIVE BENDING RIGIDITY FOR STEEP TAPER ANGLES

It is observed from Eq. (9), that the reduced elastic stiffnesses $\tilde{Q}_{11}$ of the facing will become negative if the taper angle ϕ is larger than a critical taper angle $\phi_{cr} = \frac{1}{2}\cos^{-1}(2\nu - 1)$. Since the critical taper angle depends only on the Poisson's ratio of the facing material, it lies in the range $45° < \phi_{cr} < 90°$. If the tapered sandwich member is symmetric and the taper angles is larger than ϕ_{cr}, the bending rigidity D_{11} is also negative. This implies that a positive bending moment M_x would cause a negative mid-surface curvature κ^0_{xx}. Although this behavior is counterintuitive, it has a physical explanation. For a tapered cross-section subjected to a positive M_x, the in-plane strain ε_{ss} in the facing is positive, but the Poisson's contraction ε_{nn} is negative. The negative Poisson's contraction of the facing, in conjunction with the steep taper angle, would result in a negative axial strain ε_{xx}.

RESULTS AND DISCUSSION

We present results for a tapered sandwich beam composed of a honeycomb core and isotropic facings in cylindrical bending. The primary objectives are 1) to compare the results of the analytical model and finite element analysis for symmetric tapered sandwich sections; 2) to understand the influence of the bending-shear elastic coupling on the deformation; 3) to validate the analytical model for steeply tapered sandwich beams with negative bending rigidity; and 4) to develop a further understanding of the response of tapered sections.

Table 1 Comparison of the analytical results with FEA for $H = 1.8$ m, $t = 0.8$ mm and $\phi = 60°$at point S

Variable	Analytical model	ABAQUS FEA	Error
σ_{xx}	-1.673 MPa	-1.671 MPa	0.12 %
σ_{zz}	-5.019 MPa	-5.026 MPa	0.14 %
τ_{xz}	2.898 MPa	2.897 MPa	0.03 %
σ_{ss}	-6.692 MPa	-6.697 MPa	0.07 %
ε_{xx}	10.172μ	10.235μ	0.62%
ε_{ss}	-85.187μ	-85.250μ	0.07 %
$u_z(0,0)$	-8.548(10^{-5}) m	-8.509(10^{-5}) m	0.46 %

The geometry of a straight symmetric tapered section at a constant taper angle ϕ is shown in Fig. 4. The member is simply supported on the edges at $x = \pm L$. The facings are made of Aluminum of thickness $t = 0.8$ mm. The material properties of Aluminum are taken as $E = 70$

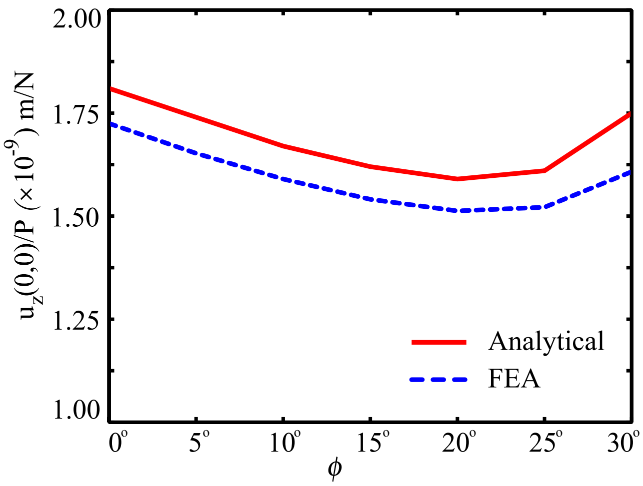

Figure 5 Comparison of analytical and FEA deflection for various taper angles ϕ; $H = 60$ mm, $L = 45$ mm and $t = 0.8$ mm.

GPa and $\nu = 0.33$. The honeycomb is also made of Aluminum (Hexcel HexWeb^{TM} 5052 4.5-1/8-10) with shear moduli $G^c_{xz} = 483$ MPa and $G^c_{yz} = 214$ MPa. The concentrated load P applied at the midspan is 10 kN. The results of the analytical model are compared to the finite element analysis (FEA) in Table 1 for taper angle $\phi = 60°$ which is greater than the critical taper angle $\phi_{cr} = 54.93°$. The plane strain FE analysis is performed using ABAQUS/Standard finite element package with approximately 30,000 8-noded quadratic plane strain elements. The analytical and FEA results are compared at point S that is located at the top surface of the upper facing at $x = L/2$, as depicted in Fig. 4. The numerical comparison in Table 1 shows that results from the analytical model are in very good agreement with the plane strain FE analysis. The strain ε_{xx} at S is positive although the bending moment M_x at that location is negative. As discussed earlier, this behavior is due to the negative bending rigidity D_{11} at large taper angles.

The midspan deflection versus the taper angle is shown in Fig. 5. It should be noted that the thickness of the sandwich member at midspan is kept constant as the taper angle is varied from 0° to 30°. It is observed that the deflection decreases as the taper angle increases, until an angle of approximately 20°. This is because the facings carry some of the transverse shear loads, thereby reducing the transverse shear load acting on the core, resulting in smaller shear deformation. Beyond 20°, the reduction in cross-sectional height leads to a dramatic increase in the deflection due to the flexural deformation. For this case, the difference between the analytical and finite element deflection is approximately 5 %.

The axial variation of the longitudinal stress σ_{ss} in the facing at the bottom surface $z = z_1(x)$ of the sandwich member is depicted in Fig. 6a for taper angle $\phi = 20°$. The shear stress τ_{ns} at $z = z_2(x)$ between the core and bottom facing is shown in Fig. 6b. The analytical and FEA results are good in good agreement except at the midspan, where the stresses obtained from FEA are very large, even possibly singular, due to the abrupt change in taper angle of the facings. The analytical solution predicts that the shear stress between the core and the facing is largest at the edge $x = L$, which could cause the facing to delaminate from the core if the applied load is large.

CONCLUSIONS

We have developed an analytical model for tapered sandwich members consisting of a honeycomb core and isotropic facings. The analytical model predicts bending-transverse shear and extension-transverse shear elastic couplings due to the participation of the facings in resisting transverse shear loads. Good correlation is shown between the analytical model and plane strain FEA.

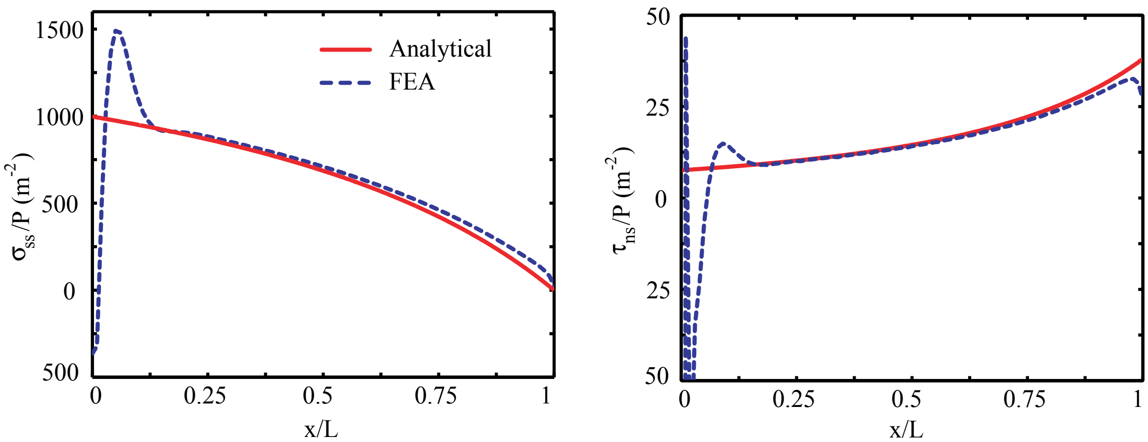

Figure 6 Comparison of analytical and FEA a) normal stress σ_{ss} at the bottom surface of the lower facing and b) shear stresses τ_{ns} at the interface between the bottom facing and core; $H = 60$ mm, $L = 45$ mm, $t = 0.8$ mm and $\phi = 20°$.

ACKNOWLEDGEMENTS

The authors thankfully acknowledge the support of the Maine Space Grant Consortium and NASA under Grant No. EP-02-01. The assistance of NASA engineers Tammy Gafka, Christopher Madden, and Ron Baccus is much appreciated.

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