My name is ______________________________

SBS, Design of Organisms: Momentum, Mass and Information Transfer 11 October 2006

Midterm I

General instructions: Make sure that you answer each part of each question. To help you in this task, I have numbered (parenthetically) each of the parts. You are free to consult the text (Vogel 1994), but not your notes. Please write your answers on the sheet with its corresponding question; use the back if needed. Please put your name on each sheet because I separate the sheets.

1. A Reynolds number, *Re*, is normally defined as

$$Re = \frac{\rho u L}{\mu} = \frac{u L}{\nu} . \tag{1}$$

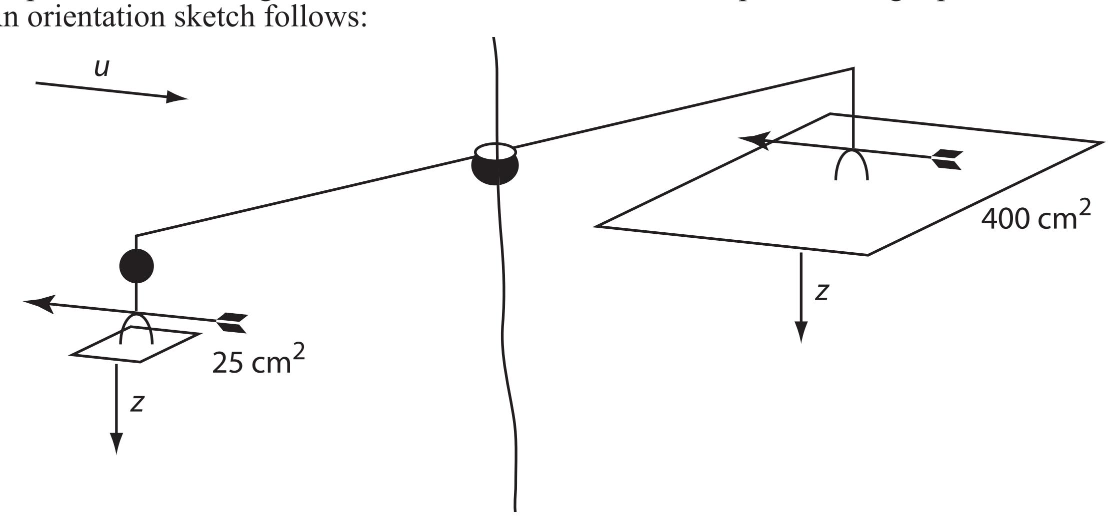
(1) For each term in either (not both of) the fractions, explain the meaning of the term and (2) give its dimensions. (3-4) Give specific examples of two kind of Reynolds numbers, (4-7) specify how and where u and L would be measured or estimated, and (8-9) explain why each kind of *Re* is useful. (10) Also explain why Reynolds numbers are useful in general. Don't get confused because Vogel uses an upper-case U whereas I use the oceanographic convention of a lower-case u.

2. Bernoulli's law can be written as

$$\Delta\left(\frac{\rho u^2}{2}\right) + \Delta(\rho g z) + \Delta p = 0 . \qquad (2)$$

(1-3) Explain each of the three terms and (4) state in words what the law states and implies. Consider a cylinder in a steady, uniform cross flow (not, say, flow in a boundary layer, where velocity varies with height above the bed). The flow is horizontal, allowing you to disregard one of the three terms. (5) Which term can you disregard and why? If this simplified law (that assumes conservation of momentum and the absence of any viscosity effects) described the flow around the cylinder fully, (6) what would the pressure distribution look like around the cylinder and (7) why? (8) How does the pressure distribution in a real (cross) flow around a cylinder differ from this idealization and (9) why? (10) What are the consequences for form drag of the differences between this idealized case and reality? This question is very relevant to biology, as many suspension-feeding appendages can be considered to approximate cylinders in cross flow, and drag determines the cost of active pumping or the cost of structural materials needed to resist strain or fracture.

3. A set of very thin, flat, rigid, glass plates of two different sizes is placed on moorings to look at settlement by fouling organisms. They are placed far enough apart to prevent flow on one plate from influencing flow on another, and a vane (like a weather vane) is mounted on top of each plate, but each plate is cantilevered out from the mooring line far enough so that the mooring itself does not obstruct the flow. The plates are all square, and the vanes arranged so that two edges of the square remain parallel to the flow (and hence to *x*), whereas two are perpendicular. The plates are also parallel to the seafloor (the *x*-*y* plane). The edges perpendicular to the flow are parallel to the cross-stream axis, *y*. The idea behind this design is to test how species diversity varies with the size of the plate (area), and the investigator has assumed that all else is equal about the environments. Two plate sizes are used, 25 cm^2 and 400 cm^2. The investigator is counting only bacteria (about 1 μm in diameter) on subsampled quadrats on the plates because she has automated means to count and group them by taxa, and so bacterial "body" size is much smaller than any one of her plates. The free-stream flow speed (and the velocities are pretty uniform and steady) is 0.5 m s^{-1}. First, (1) describe the boundary layers that will form on the undersides of the plates (the sides unaffected by the vanes) and (2) what causes them to form. (3) Are the environments identical except for total area, and (4) why or why not? Describe both the (5) vertical (*z* direction) and (6) horizontal (*x* and *y* directions) structure of the boundary layers. Do not concern yourself with diffusive sublayers (as we have not yet covered mass transfer), but do consider: (7) where you might find the outer edges of the boundary layer that will form on the undersides of the plates (not the top in this upside-down flow regime); (8) whether and where a log layer might be found; and, (9) where and whether a laminar or viscous sublayer might be present. There is no attempt to mislead you in this detailed description of a simple flow geometry. The *x* and *y* axes are exactly as we have treated them. The only departure from the typical flow geometry used in lab and in Vogel is that the boundary layer of interest is on the underside of the plate instead of on top. To avoid problems with conventions, use only positive *z* distances, where zero is the origin at the plate surface, and greater downward distances from the plate are larger positive.numbers. An orientation sketch follows:



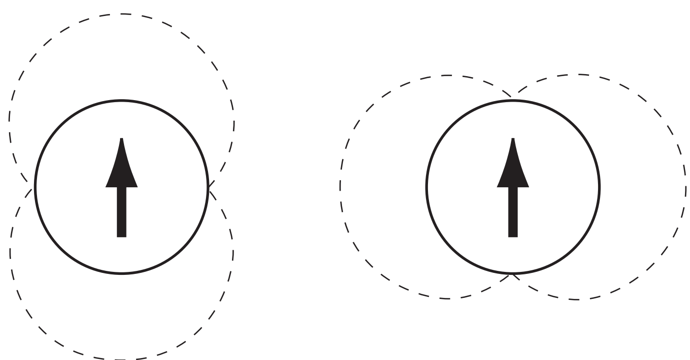
4. Consider a *rising* sphere at low *Re* as indicated by the arrow on the sphere below. Its outer surface is rigid; a bubble would rise faster because the air in it can circulate, and thus it does not follow the no-slip condition or Stokes' law. To behave this way, the sphere must be less dense than the water it is in. You may find one or more of the following equations useful:

$$w_s = \frac{2r_0^2(\rho_s - \rho)g}{9\mu}; \quad (3)$$

$$F_d = -6\pi\mu r_0 w_s\text{; and,} \quad (4)$$

$$F_g = \frac{4}{3}\pi r_0^{\ 3}(\rho_s - \rho)g. \quad (5)$$

All these equations are accurate for $Re << 1$ and reasonably accurate for $Re < 1$. The symbol w_s stands as usual for settling velocity (positive downward), so in this case it would need to have a negative value to be compatible with the observation that the sphere is rising. The force of gravity is still downward, but the net force, F_g, on the sphere due to the effects of gravity on both the fluid and the sphere is not. (1) What force or forces accelerate the sphere and (2) in what direction? (3) What two kinds of stresses decelerate the sphere? Recall that a stress is a force per unit of area. The diagram below indicates the direction in which the sphere is rising (arrow). Dashed lines indicate the magnitudes of each of the two kinds of stresses; the farther the dashed line is from the surface of the sphere, the larger that kind of stress. (3-4) Label the two (left versus right) panels with the correct labels for the stresses that they represent. (5-6) At several points around each panel draw arrows representing the directions in which those stresses act on the sphere. (7) Of the total drag on the sphere at low *Re* (Eq. 4), which kind of stress accounts for a greater proportion of the drag? (8) In what direction is the sphere moving water and (9) by what mechanism(s)? (10) Using only the best choice of terms from Eq. 3-5, write a *Re* to characterize the flow regime around the sphere.



5. The following equation applies to Newtonian fluids at low *Re:*

$$\tau = \mu \frac{\partial u}{\partial y}. \tag{6}$$

(1-4) Explain each of the terms τ, μ, ∂u and ∂y. Use any combination of words, diagrams and equations to express the concepts. For each term (5-8) give its dimensions and (9) show that the equation is homogeneous dimensionally (has the same dimensions on both sides of the equality). (10) Suggest at least one common fluid dynamic setting in which this formula can be applied, and (11) explain the relevance to one or more oceanographic or biological processes.