

FEEDBACK ON MIDTERM I

1. (1&2). Density, ρ , of the fluid is its mass per unit of volume [$M L^{-3}$]. Dynamic viscosity of the fluid, μ , is its resistance to deformation, *i.e.*, the proportionality between stress and rate of strain when the stress is kept constant and the flow is laminar. Its dimensions are not intuitive but must be the same as those of the numerator [$M L^{-3} L T^{-1} L = M L^{-1} T^{-1}$]. Be careful to distinguish dimensions from units (*e.g.*, length from meters). Kinematic viscosity, ν , is the ratio μ/ρ . You might notice that it combines the properties of the fluid into one number. You will see that it is more useful than it looks when we cover diffusion because it has the same dimensions as a molecular diffusion coefficient [$L^2 T^{-1}$] and is the diffusion coefficient for momentum. Lower-case l is a length scale appropriate to the flow geometry [L], and u is the mean or other characteristic velocity. (3,4&5). In a body Re , (of which spheres in uniform flow and cylinders in cross flow are classic examples), u is the difference between the velocity of the body and of the flow distant from the body, and l is either the characteristic distance that the flow must get diverted (*e.g.*, radius), or the square root of the cross-sectional area of the object perpendicular to the flow. For a given object shape and orientation to a uniform, steady flow, the same flow pattern will always arise at the same Re . In a pipe Re , u is volumetric flow per unit of time divided by cross-sectional area of the pipe, and l is the pipe radius (or diameter). The Re will determine whether the mean flow profile is parabolic (for low Re) or flatter in the middle (for high Re) and will help determine how much pressure drop there is along the pipe. In a roughness Re or Re_* , u is the shear velocity or u_* that is related to bed shear stress and the velocity profile, and l is a characteristic roughness height, z_0 , or grain diameter. One velocity is not enough to characterize a boundary layer, but one Re_* is. A local Re or Re_x characterizes a gradient in boundary layer structure with distance x downstream of the start of a flat plate. The velocity scale is the free-stream velocity u , whereas the length scale is x , the distance downstream from the leading edge. Depending on the smoothness of the boundary, various formulas are available to characterize the flow (*e.g.*, the boundary-layer thickness) as a function of x . (6). So long as the appropriate Re is kept the same, the flow patterns will be the same. This effect of dynamic similarity allows one to make scaled-up or scaled-down models for experimentation.

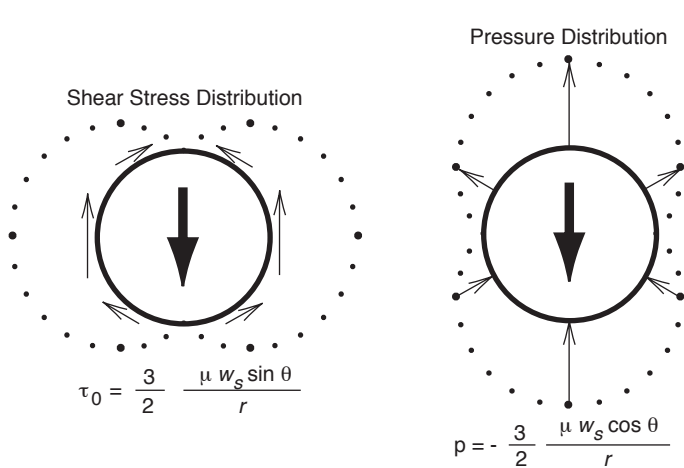
Reminder: Re characterizes the flow, not the fluid or the object (but their interaction as expressed in the flow). At high Re much of the affected volume of the flow will be downstream of the object. Re means Reynolds number, so don't write " Re number."

Extra info: I did not introduce it, because it would be confusing, but there are Reynolds numbers for flows where no object is involved explicitly. One takes l as the distance over which velocity changes by the quantity u . Physical oceanographers often use this kind of Re . We have treated only a few of the infinite variety of Re , so don't lapse into using a Reynolds number without specifying what kind and what the explicit length and velocity scales are.

2. (1) Streamlines are fairly laminar on the front half of the cylinder; the major difference as flow velocity increases is greater streamline compression in going around the front half. An attachment point should be shown on the midline of the front of the cylinder, and separation points should be shown someplace behind the location of maximum width (y dimension). *The oncoming flow was specified to be uniform, so there is no horseshoe vortex.* What produces the horseshoe vortex is an impermeable bed and a gradient in velocity above it. Flow is slowest (zero) at the stagnation point (attachment point) and at the detachment points, although you could correctly say that flow velocity is zero everywhere on the cylinder's surface. (3) Flow is fastest at the location of maximal width (both sides). (4) The streamline upstream of the attachment point and streamlines downstream of the detachment points are the stagnation streamlines. (5) Downstream of the separation points, the flow varies with time, and so do streamlines. (6) Pressure is highest on the attachment point, where the flow is stalled and lowest at the sides of the cylinder where flow has been accelerated beyond the oncoming, uniform velocity. (7) Where

flow is stalled, kinetic energy is converted to pressure, which in turn forces the flow around the object. (8) There is little difference on the front and up to the widest (y direction) of the part of the object because the flow is following a similar shape. (12) Flow past that point is slowing because the body is becoming narrower, but because it narrows gradually the flow slows without separating until very near the end, so more of the kinetic energy is regained as pressure rather than lost to turbulence. (Admittedly, some is lost to skin friction.) (9) So the biggest pressure difference between the cylinder and streamlined body are over the rear. (12) The streamlined body therefore has less form drag. (10) There is little recirculation behind the streamlined body (reversed direction of flow behind the separation points), so there is a large difference in velocity toward the rear as well. (11) The more gradual taper of the streamlined body means less loss of energy to turbulence. As the flow slows along the back end due to the increased volume of fluid past the widest point, a greater fraction of the kinetic energy is returned to pressure that is near the surface and pushes on it. This pressure is perpendicular to the surface, so it has an upstream component. You have added to skin friction, but since drag is lower overall, you must have decreased form drag.

3. (1) The gradient in velocity in the boundary layer tilts the alga. The combination of float and stipe form an angle to the horizontal. Lift pushes that combination downward. There will be a little lift upward, on the fronds. (2) Drag pulls all the parts downstream, but is stronger where the flow is faster and the alga's area is greater. (3) There is no source of thrust in this system unless we pop a float. (4) At zero flow velocity tension will be straight up and equal to the difference in mean density of the whole plant from seawater times plant volume times g. So that's $[M L^{-3}]$ times $[L^3]$ times $[L T^{-2}]$, which gives $[M L T^{-2}]$, the dimensions of the tensile force (upward). This force will continue, but the drag and lift will be added as velocity is increased. The net tension (pull) on the holdfast will be at an increasing angle to the vertical as flow velocity increases. (5) The magnitude of the drag will increase with flow velocity initially and as long as velocity continues to increase. It won't increase as much, however, as if the alga remained rigid. (6) The drag coefficient will decrease as the alga bends over and gets more streamlined. It may go through an episode of increase if flapping occurs in some particular range of flow speeds.



4. (1) A settling sphere is accelerated downward by gravity in proportion to sphere excess density and volume and decelerated by net upward pressure and shear forces. The gravity force operates on the whole sphere (and is called a body force for that reason.) Note that the arrows for shear are tangential to the sphere, and the arrows for pressure are perpendicular. (2) Drag is exactly balanced by acceleration due to gravity. (3) Because acceleration depends on radius cubed and drag depends on radius, the net dependence is on radius squared. It's not intuitive.

5. (1) The abbreviation u is used for horizontal, downstream, velocity. (2) The abbreviation z is used for the vertical component of Cartesian coordinates. (3) The velocity, u , is a vector with mean direction as shown by the arrow on the graph itself. The coordinate z is also a vector since it carries information on both distance and direction (*i.e.*, is positive upward as shown by the arrow to the left of the axis label). (4) The viscous sublayer shows a linear gradient in flow

velocity and laminar flow. (5) This transitional region shows periodic variations in velocity and is intermediate in character between laminar and turbulent flow. (6) The log layer is turbulent, with velocity varying in direction and magnitude. (7) At any instant, the flow at some spot in the log layer could be going in any direction. (8) Because of the no-slip condition, velocity has to be zero at $z = 0$. (9) Any value that was not gigantic was OK as long as you used a linear scale. The actual values shown are pretty close to the average flume conditions that we have used.

