

## Feedback on Midterm 2

I was fairly happy with the results of the exam. Most of you “got it.” One common problem, however, was telling me everything you could think of about a problem, whether or not those aspects had any relevance to the question. Several of you also answered parts of questions that were no longer on the exam. Answer the one asked, not the one you reviewed. For example, several people spent half their space on inertial impaction and gravitational deposition before they mentioned that these mechanisms required excess density and hence were irrelevant to the question posed. Conversely, make sure that you answer all the parts. Several people failed to answer part (2) of question 1. For the remaining study question and final, please don't diffusively deposit prose.

Many of you are confused about the difference between particles and dissolved molecules, and it affected several of your answers. Dissolved molecules are kept in solution by thermal motion of water molecules. They exist as single molecules surrounded by water and if they are small have about the same diffusion coefficient as that of water itself ( $10^{-9} \text{ m}^2 \text{ s}^{-1}$ ). Particles typically contain many orders of magnitude more molecules. Even clay grains (really flakes since most clay minerals form plate-shaped crystals) that are  $< 2 \mu\text{m}$  in diameter will settle out by gravity in perfectly still water. They are particles (as are silt and sand). The dominant mineral on earth is silicon dioxide ( $\text{SiO}_2$ ). A mole ( $6.02 \times 10^{23}$ ) molecules of  $\text{SiO}_2$  has a mass of about  $28.086 + 2(15.9994)$  or about 60 atomic mass units. The most common form of silicon dioxide is quartz, with a density of about  $2650 \text{ kg m}^{-3}$  (or  $2.65 \text{ g cm}^{-3}$ ). One mole of quartz thus weighs about 60 g. If a cubic centimeter of quartz weighs 2.65 g, a cubic micrometer would weigh  $2.65 \times 10^{-12} \text{ g}$ . A particle this small still contains about that weight divided by 60, or  $4.4 \times 10^{-14}$  moles of silicon dioxide. That is still about  $2.7 \times 10^{10}$  molecules (*i.e.*, 27 billion)! A particle is vastly different than a molecule. Colloids (particles with no dimension greater than about  $0.1 \mu\text{m}$  and usually with all of their dimensions considerably smaller) bridge the gap between solutions (dispersions of individual molecules in a solvent) and suspensions. They can be held in suspension by thermal motion of water, but in still water will form concentration gradients. The fuzzy part of the difference between molecules and colloids arises for polymers, whose chains can reach huge molecular masses. “As with solutions, colloids may exist in any one of the three physical states of matter. These different types of colloids have different names: a foam is a gas dispersed in a liquid or solid, a liquid aerosol is a liquid dispersed in a gas; a solid aerosol or a smoke is a solid dispersed in a gas, an emulsion is a liquid dispersed in a liquid, a gel is a liquid dispersed in a solid, and a sol is a solid dispersed in a liquid or solid.” <http://bengu-pc2.njit.edu/trp-chem/chemistry/Solutions/Col.html> So that's why we call it hydrosol filtration theory.

Several of you tried to apply hydrosol filtration (feeding on suspended colloids) theory to osmotrophy (feeding on solutes). Only molecular diffusion works for the solutes. There is no organized flow at the scale of individual molecules to provide direct interception or inertial impaction. Remember that molecules are moving at  $600 \text{ m s}^{-1}$  in all directions. They won't fall out of solution by gravity. The Brownian diffusion mechanism for hydrosols is analogous to the molecular diffusion mechanism for solutes and the latter is the only one that will work for solutes.

1. The problem set the axis of the cylinder perpendicular to both the gravity vector and the mean flow velocity,  $u$ . The flow could have any angle with respect to gravity; but you all chose  $u$  to be perpendicular to gravity. Several of you confused the orientation of the cylinder. If its axis is perpendicular to gravity, it not oriented “up and down.” Because the particles have no excess density, there would be no difference in particle encounter as flow direction changed with respect to gravity, however. Gravitational deposition and inertial impaction depend on excess density, so they do not occur at all, and dealing with them was not relevant. Several of you thought that gravitational deposition would work. Look back at Stokes' law and see that it would not. Finding  $\rho_s = \rho$  is an “unsettling” result. I did not specify a cylinder  $Re$ , so the exact geometry of encounter would be difficult to specify. Direct interception would occur primarily on the upstream half of the cylinder, up to the point where flow is deflected the farthest and streamlines are compressed the most, with possibly some additional encounters in a recirculation-reattachment region at the rear at intermediate and high  $Re$ . Diffusional encounter by the smaller particles (about  $2 \mu\text{m}$  and smaller) would occur over the entire cylinder, but at a lower rate in the thickest part of a particle-depleted boundary layer, whose thickness and location would also change with  $Re$  and with the ability of the collector to hold onto them and thus remove them from suspension and decrease the local concentration at the surface ( $C_0$ ).

2. Terminology got in the way of many of your answers. Looking back at my lectures, I gave you in the early BBL lecture a slide with the terms hydraulically smooth and hydraulically rough. Hydraulically smooth and smooth turbulent mean the same thing, as do hydraulically rough and rough turbulent. I mentioned that once but did not write it down or put it on a slide. In smooth-turbulent boundary layers there are viscous and diffusive sublayers; in rough-turbulent boundary layers there are not. Because I never gave you anything printed with “smooth turbulent” on it, I graded your discussion of this terminology very leniently and graded the question overall very leniently. The horizontal flow affects the vertical flux and of solute only by affecting the thickness of the diffusive sublayer, which changed from 2 mm thick in the original flow to 4 mm thick. Flow must have slowed for the solute gradient to change from the dashed to the solid line. The gradient halved in steepness, from  $8 \text{ mol m}^{-3}$  in 2 mm to  $8 \text{ mol m}^{-3}$  in 4 mm. The vertical flux is down gradient and so is upward, with its magnitude also halved after the slowing, since the gradient was cut in half but neither  $D$  nor  $S$  changed. A couple of you tried going part way toward flux calculations but forgot to convert everything to the same units. If I plug in the numbers (Eq. 2 on the test) for the original, faster flow, I get (dashed line):

$$Q = -D \frac{\partial C}{\partial z} S = -2 \times 10^{-9} \text{ m}^2 \cdot \text{s}^{-1} \left( \frac{8 \cdot \text{mol} \cdot \text{m}^{-3}}{2 \times 10^{-3} \text{ m}} \right) S = (-8 \times 10^{-6} \text{ mol} \cdot \text{s}^{-1} \cdot \text{m}^{-2}) S. \quad (1)$$

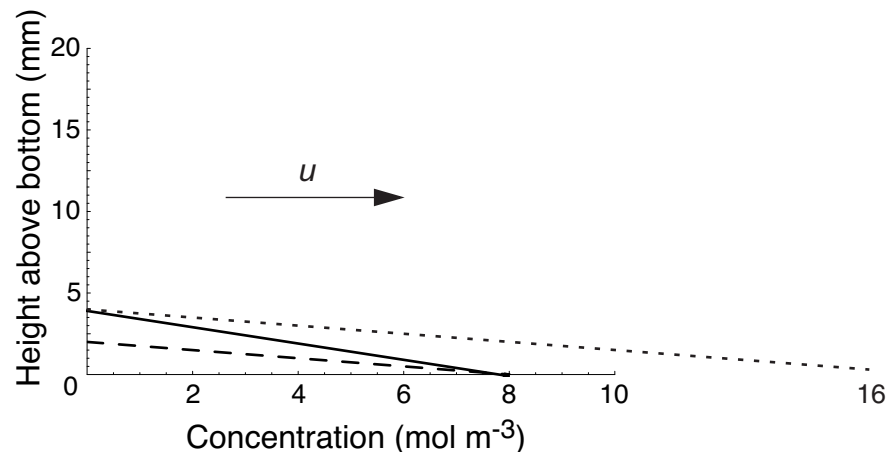
For the slower flow, I get:

$$Q = -D \frac{\partial C}{\partial z} S = -2 \times 10^{-9} \text{ m}^2 \cdot \text{s}^{-1} \left( \frac{8 \cdot \text{mol} \cdot \text{m}^{-3}}{4 \times 10^{-3} \text{ m}} \right) S = (-4 \times 10^{-6} \text{ mol} \cdot \text{s}^{-1} \cdot \text{m}^{-2}) S. \quad (2)$$

For the dotted line, I would get:

$$Q = -D \frac{\partial C}{\partial z} S = -2 \times 10^{-9} \text{ m}^2 \cdot \text{s}^{-1} \left( \frac{16 \cdot \text{mol} \cdot \text{m}^{-3}}{4 \times 10^{-3} \text{ m}} \right) S = (-8 \times 10^{-6} \text{ mol} \cdot \text{s}^{-1} \cdot \text{m}^{-2}) S \quad (3)$$

As an aside or extension of the problem, the billion bacteria in each milliliter of pore water would probably keep making whatever they were making at nearly the same rate for a while, but it is no longer getting carried away as fast, so it will build up in the pore water (dotted line). To keep up the same flux (to match a constant rate of production) would entail a doubling of the concentration (dotted line), matching the slope of the initial gradient and thus the initial flux. It's the gradient, not the concentration or distance alone, that drives the flux.



3. A couple of you thought of  $D$  as a variable, which got you seriously confused. It is the molecular diffusion coefficient, which is a constant for a particular (dilute) molecule in solution at a given temperature. The graphs represent the *average* time that it would take a molecule to go from point A to point B if A and B are separated by the straight-line distance indicated. The plots in questions 2 and 4 and the plot plots for question 3 are completely compatible. All are true at the same time. Plots like the one in question 4 can be confusing in this regard because the concentration profiles go out to different absolute distances depending on the radius of the cell,  $r_0$ .

That is, for a value of 2 on the abscissa, the real distance is 1  $\mu\text{m}$  for a 0.5- $\mu\text{m}$  (radius) bacterium but 200  $\mu\text{m}$  for a 100- $\mu\text{m}$  phytoplankter. Flux in the radial direction depends on the gradient in absolute dimensions,  $r$ ; and not relative dimensions,  $r/r_0$ , so the bigger the cell, the lower the flux per unit of area of cell surface because the lower the actual gradient. Most of you focused correctly on thickness of a concentration boundary layer in setting the typical diffusion time and hence flux magnitude for a given concentration difference across this boundary thickness. One example is the thinning of the concentration boundary around the anterior and sides of a swimming, large phytoplankter affording a larger net flux of nutrients (higher  $Sh$ ). Extreme examples of large organisms feeding by diffusion are hydrothermal vent worms (vestimentiferans) that also take advantage of the flow, in this case produced by hydrothermal circulation, over the extensive surface area of their exchange organs or plumes. Respiratory systems of large organisms differ but themes are similar. Fish drive water through their gills to benefit from both short diffusion distances and large surface areas. Another example is the ability of small organisms ( $< 100 \mu\text{m}$  or so in smallest dimension) to respire without an elaborate respiratory and circulatory system. You and I can't wait long enough for diffusion to work over 2 m.

4. The solid line is correct with the corrected label. You all correctly surmised that concentration must fall with distance from the cell (because it is a source) and most of you picked the correct, solid curve. The flux to the cell and flux through successive spherical shells must be the same at steady state (or else concentration would fall or build up at some distance,  $r$ . The flux away from the cell per unit of area therefore must decrease with distance from the cell in exact proportion to the surface area, which increases as  $r^2$ . Nothing else in Eq. 2 changes, so if  $Q$  is constant then the *gradient* must vary as  $1/r^2$ ; it is much steeper near the cell than far away. The dashed line would apply if the cell were a sink (taking up the compound), and these same arguments apply. More intuitively, the volume of a spherical region expands so greatly with increasing  $r$  (as  $r^3$ ), that the cell can have little effect far away, and so there can be little change and no steep slopes in concentration at great distance. Therefore both the dotted and the dash-dotted lines must be erroneous or due to some unsteady, dramatic event. Diffusion would quickly eliminate such a steep gradient at 14-15 cell radii from the cell's center. Likewise, diffusion is pretty efficient at taking away or delivering material at small scale, but a gradient as flat as that near the cell in the dotted and dash-dotted cases would be hard to achieve except by advection. Please notice that the gradients in the dashed line are identical in magnitude with those of the solid line, only opposite in direction. The same is true of the other two lines.
5. Most of you focused appropriately on thinning of the diffusive sublayer. Several of you tried to suggest that you would have a broadly distributed, uniform flow directed either straight downward or at an angle to the vertical. Kinetic energy would be converted to pressure, which would then either push the flow horizontally along the surface or make the flow stop (depending on what happened further out horizontally). The flow can have no vertical component at the bed, since it can't go through it. I fixed total volumetric flow rate but did not specify it; you might or might not be able to generate a turbulent flow with this volumetric flow rate directed horizontally along the surface. If you had one big jet coming down in the middle, you would greatly thin any diffusive sublayer in the fast flow at the center, but the radially spreading flow would rapidly lose speed because it is covering more area and volume as it spreads radially. Because of this radial deceleration, several of you chose to have flow come from the side in the usual orientation of  $u$  perpendicular to  $z$ . Thinned regions are disproportionately important, however, analogous with short circuits. One of you suggested making one big, central, perpendicular jet turbulent, but recall that turbulence uses up energy that might have been used in thinning the diffusive sublayer; also shooting a jet at a surface will result in turbulence anyway, so making it turbulent ahead of time doesn't help. A better idea that several of you had was to distribute jets or roll vortices by shooting distributed jets or sheets of water at the bed. Each sheet or jet would cause extreme thinning of any diffusive sublayer. A return (upward) circulation would be produced between the sheets or jets, removing the depleted water, too. If you wanted to get real fancy, you could have multiple jets and constantly move them, so a diffusive boundary layer would not have time to form anywhere. (That's an unsteady-state solution.)