

REVIEW OF TERMS AND CONCEPTS

Words and manipulation of dimensions: If you understand manipulation of dimensions, you don't need to memorize so many words.

$$F = ma; \text{ Force} = \text{mass} \times \text{acceleration} [\text{M L T}^{-2}]. \quad (1)$$

$$\text{Work or energy} = \text{force} \times \text{distance} [\text{M L}^2 \text{T}^{-2}]. \quad (2)$$

Shear and pressure are forces per unit of area $[(\text{M L T}^{-2})/\text{L}^2 = \text{M L}^{-1} \text{T}^{-2}]$. No matter what the angle of the force that the fluid exerts at a point, you could decompose the force into a shear and a pressure component, just as you can decompose any vector in a plane into orthogonal x and y components. Both pressure and shear are vectors, but the pressure in a fluid has the same magnitude in all directions at any point in the fluid, whereas the shear vector at a point has direction. The pressure on the upstream face of an object pushes it downstream because the object is facing it, not because the pressure is pushing selectively in one direction. Pressure is pushing just as hard upstream. Pressures and shear stresses when integrated over the entire surface of a body equal the total force exerted by the fluid on that body. Remembering that fact when we get back to lift can avoid lots of confusion.

A stress is a force; a strain is deformation of a material in response to that force. In a fluid, a constant force produces a constant rate of strain $[\text{L T}^{-1}]$. In a solid it produces a constant strain $[\text{L}^{-1}]$.

$$\text{Momentum or kinetic energy} = (mv^2)/2; (\text{mass} \times \text{velocity}^2)/2 [\text{M L}^2 \text{T}^{-2}]. \quad (3)$$

First, let me comment on the dimensions. You know that energy and mass are interconvertible, even if you don't want to try this trick at home: $E = mc^2$. So if you can remember the formula, you can give the dimensions for energy: A mass times a velocity squared (c being the speed of light) is $[\text{M} (\text{L}/\text{T})^2] = [\text{M L}^2 \text{T}^{-2}]$. But why does the equation for momentum or kinetic energy of water or mass of any other sort have a denominator of two? Start a body from rest; kinetic energy = 0. Now apply a force over some time, t , and you will accelerate the body to some velocity v . By definition, the object's $a = (\text{change in speed}/\text{time over which speed was changing}) = v/t$. So you already know that $F = mv/t$. The average speed of the object over t is simply $v/2$. What distance did it go over t if its speed is $v/2$? Simply multiply the speed times the time to get the distance (check dimensions): $vt/2$. Work and energy have the same dimensions. Now multiply the force times the distance:

$$\frac{mv}{t} \times \frac{vt}{2} = \frac{mv^2}{2}. \quad (4)$$

If you divide momentum by volume you get momentum per unit of volume $[(\text{M L}^2 \text{T}^{-2})/\text{L}^3 = \text{M L}^{-1} \text{T}^{-2}]$ for a fluid moving with the specified velocity. In terms of variables that you see in the book (Eq. 4.2, p. 53), momentum/volume = $\rho U^2/2$. Notice that the dimensions are the same as those of pressure, so you get a sense that a pressure as a kind of potential energy in a volume is convertible into a velocity of that volume of fluid as kinetic energy. Bernoulli's law is an expression of this interconvertibility; for the usual, frictionless (inviscid) version of the law, the interconversion is perfectly efficient.

Pressure at the front of an object (that is moving relative to the water) is high because momentum has been extracted from the fluid and converted to pressure. If you always worry about the *relative* flow velocity, you will not confuse the front (upstream) and back (downstream) faces of a body. Pressure drops at the sides because kinetic energy is gained at the expense of pressure (so the pressure does not continue to rise at the front, but instead drives the flow around). Pressure in a real fluid fails to return to its frontal maximum at the rear because friction extracts energy and because, as you saw most clearly for very low Reynolds numbers, a departing fluid that has viscosity pulls on the object (exerts tensile stresses on the object in the form of low pressure). Above very low Re , the wake is in general less organized than the oncoming flow (having higher entropy, with turbulence being dissipated to laminar shear and in turn to heat).

In a real fluid and the real plumbing of your home or irrigation system assume that the pipes all are horizontal, except for the inflow from a head tank and that all pipe diameters are the same and constant. Set up an initial flow rate under a constant head of pressure. Now keep that (driving) head of pressure constant and add more horizontal pipe. The flow rate out the end of the pipe (and everywhere else in your pipe) will decrease with the added length of pipe because you are adding frictional losses to the system but not adding any driving force. The difference in pressure between the inlet and outlet ends of your pipe will therefore increase. Let's say that as a member of the volunteer fire department you find this reduced rate of flow unacceptable. To return the flow rate to its original, you will have to raise the head tank. That is, if you keep the head of pressure the same, in a longer pipe you will get a slower flow rate. Conversely, if you want to keep the flow rate the same, you will need a higher head of pressure. Exercise your understanding by putting these ideas together with the diagram (Fig. 4.2) on p. 53 of your text. Fig. 4.2 does not suggest any frictional losses.

Distinguish properties of fluids from properties of flows. Dynamic and kinematic viscosities and densities are properties of the fluid, whereas Reynolds numbers are properties of flows. Unless you specify otherwise, I will assume that you follow the oceanographic convention that x is the streamwise dimension and u is the corresponding velocity (positive downstream); y and v are the corresponding measures in the cross-stream direction, with z and w referring to the vertical.

Uniform – unchanging in space (*e.g.*, a uniform oncoming flow impinging on a sphere having the same velocity, u at all y and z coordinates that are far enough upstream or laterally away to be unaffected by the sphere. If we are considering a benthic boundary layer, uniform generally means that the downstream velocity does not vary across the stream ($\partial u/\partial y = 0$) or with downstream position. That is, velocity varies only with z (also called a 1D flow).

Steady – unchanging in time (*e.g.*, the flow in a good, head-tank-driven flume at laminar flow velocities being unchanging in time) ($\partial u/\partial t = 0$). Note that downstream of an obstruction a flow may become unsteady. In a turbulent or transitional flow, the instantaneous velocity is not steady, but the average velocity may be.

Fully developed – generally refers to a benthic boundary layer whose thickness has stopped changing in the downstream direction ($\partial u/\partial x = 0$, as opposed to a growing or eroding boundary layer where $\partial u/\partial x \neq 0$).

Laminar flow – flow that follows contours, be they of the bed or of plates or bodies suspended in the flow, your indication that viscous forces dominate inertial ones or that the inertial forces are parallel to the body contours.

Turbulent flow – has no universally accepted definition, but is recognized by a “syndrome” of properties indicating that inertial forces have overcome viscous forces. Turbulence is:

- irregular (random, not laminar or periodic);
- diffusive (good at moving momentum, heat and mass around, but not in one direction);
- characterized by large Re ;
- characterized by 3D vorticity fluctuations (vortices of varying size and magnitude with axes in all three dimensions);
- dissipative (using up kinetic energy rapidly by converting it into small vortices that in turn send it irretrievably to viscosity and heat);
- well above the molecular scale (not to be confused with Brownian motion); and,
- a feature of the flow (not of the fluid).

Fluid dynamicists are getting braver now, however, and definitions are being proposed. Davidson (2004, p. 53) has defined hydrodynamic turbulence in an incompressible fluid as “a spatially complex distribution of vorticity which [that] advects itself in a chaotic manner in accordance with the vorticity equation. The vorticity field is random in both space and time, and exhibits a wide and continuous distribution of length and time scales.” Wu *et al.* (2006, p. 106) attribute an even simpler definition of turbulence to Bradshaw: “randomly stretched vortices.”

When visualizing a flow, consider whether you are visualizing a streamline, a streakline or a particle path.

The following equations should look familiar. Be prepared to identify their variables by name and explain the concepts behind them.

$$\tau = \mu \frac{\partial u}{\partial y}. \quad (5)$$

This equation holds over at least small scales for Newtonian fluids like water, and it says that shear stress, τ [$M L^{-1} T^{-2}$, a force per unit of area] is linearly proportional to shear. Constant shear stress in an ideal fluid produces constant rate of strain (constant $\partial u/\partial y$). Shear is a gradient in velocity ($\partial u/\partial y \neq 0$). You know that the coefficient (μ) expressing proportionality between shear and shear stress (τ) is a dynamic viscosity, but you have forgotten its dimensions. So exercising what you know about dimensional homogeneity, you (6)

$$M L^{-1} T^{-2} = [\mu] \frac{L T^{-1}}{L} = [\mu] T^{-1}. \quad (6)$$

Therefore the dimensions of $\mu = [\mu]$ must be $M L^{-1} T^{-1}$. Shear is often expressed as a shear rate, in dimensions of T^{-1} . Looking at the dimensional equations makes it more apparent why. Change in velocity has the same units as velocity and velocity per unit of distance has dimensions of T^{-1} . Eq. 5 indicates that shear stress increases linearly with both viscosity and the gradient in velocity. Since there is a no-slip condition at the boundary, it should make sense that shear stress on the bed (τ_0) increases with free-stream velocity (because turbulence only steepens the gradient near the bed).

The most important equation that you have exercised so far, in terms of providing you with insight into flows is the equation for Reynolds number, Re :

$$Re = \frac{\rho U L}{\mu}. \quad (7)$$

The reason that this equation is so useful is that for a fixed geometry of flow and fixed geometry of object, flow geometry will be similar at similar Re . For the object shape and orientation, in going from low to high values, flow will first be laminar, then begin to show some periodicity in space and time (eddying), then become turbulent. Where the transitions happen depends upon flow and object geometries and orientations. For a sphere, the transitional regime starts near $Re = 1$, and the flow in the wake is turbulent by somewhere near about $Re = 1000$. For a cylinder in cross flow, the transitions occur at Re based on cylinder diameter of about 40 and 400. Based on radius, the transitions would be near 20 and 200. For a pipe flow, the transition to turbulence usually occurs by $Re = 2300$ based on pipe diameter and mean flow velocity across the pipe, but the transitional region is small, with the flow being laminar for $Re < 2000$. So, a value of Re that is fully turbulent in one regime may be laminar in another flow-object geometry. Fully turbulent does not mean turbulent everywhere. Even though the center of the pipe and the wake of the sphere each would be fully turbulent at $Re = 5000$, the pipe would have laminar flow immediately adjacent to its walls, and the sphere would have a laminar sublayer over its upstream face (assuming that you started with a uniform oncoming flow). So don't present, accept or try to interpret a Re without knowing the geometry of the flow and the object.

The most complicated Re that you have encountered in class is the roughness Re or Re_* (also called the boundary Reynolds number). Unfortunately, there are two versions that use different length scales. The version used in your book (pp. 171-172) uses the grain diameter of a smooth sediment bed that has no heterogeneity in grain size (and no ripples or bumps, just grain roughness, described by grain diameter,

$$Re_* = \frac{\rho u_* d}{\mu}$$

If this Reynolds number is above 100 or so, there is no viscous sublayer and the flow is turbulent everywhere beyond about $3d$ from the bed; it's called hydraulically rough (or rough-turbulent). If it is between about 3.5 and 100, the flow is (hydraulically) transitional, and a viscous sublayer comes and goes at any one spot. For $Re_* < 3.5$, there is a viscous sublayer, and the flow is called hydraulically smooth (or, more descriptively, smooth-turbulent). Note that a hydraulically smooth flow contains laminar, transitional and turbulent regions, respectively, as distance from the bed increases. A more broadly useful (but conceptually a bit more complicated) length scale to replace d is the hydraulic roughness height, z_0 , that comes out of the so-called "law of the wall," which approximates the average flow velocity \bar{u} at height z as:

$$\bar{u}(z) = \frac{u_*}{K} \ln \frac{z}{z_0} \quad (9)$$

Both u_* and z_0 are found by measuring average velocity at several heights above the viscous sublayer, in the so-called log(arithmic) layer where $\ln z$ versus mean velocity plots as a straight line. The shear velocity can be calculated from the slope of that line, and z_0 can be estimated as the z intercept where mean downstream velocity would equal zero if the log layer were a good model all the way to the wall. It is a good model so close to the bed only in a rough-turbulent flow, but z_0 is a useful roughness scale even for smooth-turbulent and transitional boundary layers.

Because Eq. 9 makes it clear that velocity varies with height, no single velocity can characterize the flow in a boundary layer. You need two parameters, at a minimum one velocity and one length scale, and the Re^* provides both. Moreover u_* is a uniquely useful velocity scale because

$$u_* = \sqrt{\frac{\tau_0}{\rho}} \quad (10)$$

Drag is a force. It is the rate of removal of momentum from the flow by a solid object in the reference frame where the object stands still and the flow moves past. This reference frame does not preclude motion of the object (e.g., a fish), but then the reference frame moves along with the object. There are two different ways to divide drag in two. One is by the dividing into pressure versus shear forces. The former kind of drag is called form drag and the latter, skin friction. The other is by dividing into two terms in the formula for drag, D , where S is the maximal cross-sectional area of the object in a cut perpendicular to the direction of flow (more accurately a projection of the object onto a plane perpendicular to the direction of flow):

$$D = \frac{\rho S u^2}{2} \times k Re^a \quad (11)$$

You will recognize the first term on the right as the dynamic pressure times the projected area, which will give dimensions of a force. Think, however, about the object whose upstream face you cut off. You did not change the projected area, but you sure changed the dynamic pressure acting on the body. Let's take \sqrt{S} as the length scale for Re , which is one reasonable choice. Now

$$D = \frac{\rho S u^2}{2} \times k \left(\frac{\rho \sqrt{S} u}{\mu} \right)^a. \quad (12)$$

It looks as though we should be able to simplify because the numerators share so many terms, but we can't because a is unknown. The equation would be much simpler if the two terms were additive, but the second acts as a factor times which you have to multiply the first. For anything but very small Re , you can think of it as an inverse streamlining factor. The better the streamlining, the smaller will be a . There is no way from staring at Eq. 12 to figure out how to partition between form drag and skin friction; changing the streamlining (and therefore the behavior of the fluid at a given Re), will change the partitioning. When you see the drag formula in its usual form,

$$D = \frac{C_d \rho S u^2}{2}. \quad (13)$$

resist the temptation to think that drag is really simpler than in Eq. 12 and beware that the entire second term of Eq. 11 & 12 is equal to the drag coefficient, C_d . The drag coefficient, then, is a characteristic of the body, the flow, and their respective orientations, not just of the body.

Now, for spheres you know from Stokes' solution that $D = 6\pi\mu r_0 u$. Let's exercise what you know by looking at drag on a sphere by starting with Eq. 13, which you know to be valid and by setting it equal to Stokes' solution, which you also know to be valid so long as $Re \ll 1$. For this particular case, we can solve for the factors entering into the drag coefficient by noticing that $S = \pi r_0^2$;

$$D = \frac{\rho \pi r_0^2 u}{2} \times C_d = 6\pi\mu r_0 u. \quad (14)$$

The solution even here is not easy; $C_d = (12\mu)/(\rho r_0 u) = 12/Re$. I could use diameter instead of radius, but if I did, then C_d would equal $24/Re$. If you look closely, you will see that Eq. 11 and 14 are consistent with this solution only through k . Eq. 11 comes from dimensional analysis, and dimensional analysis admits imprecision of up to a factor of 10 or so in each term, so the factor of 12 that is needed to make the drag solution work comes as no surprise. These difficulties with a case where an exact solution is known help make clear both why drag coefficients are so much used and how they can mislead. There is much more going on with a sphere, but I'll leave it alone till later. If you need a pretty good empirical equation, see Eq. 15.2 in Vogel. I still prefer coefficient-free drag formulations. For a sphere and $Re \leq 0.7$, some theory only a little more complicated than we have treated in class yields:

$$D = 6\pi\mu r_0 u \left(1 + \frac{3}{8} Re \right). \quad (15)$$

This result is also well verified by experiment for $Re \leq 0.7$. Note that as Re gets very small, this formula converges on $6\pi\mu r_0 u$, as it must.

Why should you care about any of this stuff? It affects how much energy it takes you to swim or bicycle, how hard it is to get dirt off your car with a stream of water (that darn' no-slip condition), and the wind velocity that will rip you off your feet. Water packs a bigger wallop per unit of volume at a given velocity. Drag influences swimming performance (speed for a given, steady force) and rate of energy expenditure for a motile organism. Drag, in its steady and unsteady (to be introduced later) forms, limits sizes and shapes of organisms that project from rocks or sediments into bottom boundary layers.

Rate of fluid arrival in the vicinity of an organism has a lot to do with how much mass (e.g., potential food) arrives in its vicinity, whether the system is planktonic or benthic. So the flow rates and patterns matter in many more ways that we will visit shortly.

Readings for Exam I.

Please be familiar with all the material of Ch. 1 - 8. It will do you no harm to read Ch. 9, but it also deals with issues that we will cover under mass transfer. Although you are not responsible for the material in it (except what has been given in class on drag in boundary layers), it is good motivational reading about the biological importance of boundary layers. You are encouraged to take a look at Ch. 15, but your lectures and handouts contain more detailed information on Stokes' settling than you will find in Ch. 15, and the rest of Ch. 15 is not covered in the exam.

I will ask no specific questions that involve lift on this exam, but I will on the final. I encourage you to exercise this concept early and often to get used to it. Likewise, I will address thrust in the swimming lecture and final. **CAUTION: I often use slightly changed questions from one year to the next that change the correct answer. I am looking more for your reasoning than for one, correct answer.**

Davidson, P.A. (2004) *Turbulence: An Introduction for Scientists and Engineers*. Oxford University Press, Oxford.
Wu, J.-Z., Ma, H.-Y., Zhou, M.-D. (2005). *Vorticity and Vortex Dynamics*. Springer, Berlin.