

REVIEW OF TERMS AND CONCEPTS

Words and manipulation of dimensions: If you understand manipulation of dimensions, you don't need to memorize so many words.

$$F = ma; \text{ Force} = \text{mass} \times \text{acceleration} [\text{M L T}^{-2}]. \quad (1)$$

$$\text{Work or energy} = \text{force} \times \text{distance} [\text{M L}^2 \text{T}^{-2}]. \quad (2)$$

Shear and pressure are forces per unit of area $[(\text{M L T}^{-2})/\text{L}^2 = \text{M L}^{-1} \text{T}^{-2}]$. No matter the angle of the net force that fluid exerts at a point on an object, you could decompose the force into a shear and a pressure component, just as you can decompose any vector in a plane into orthogonal x and y components. Both pressure and shear are vectors, but the pressure in a fluid has the same magnitude in all directions at any point in the fluid, whereas the shear vector at a point has direction. The pressure on the upstream face of an object pushes it downstream because the object is facing it, not because the pressure is pushing selectively in one direction. Pressure is pushing just as hard upstream. Pressures and shear stresses when integrated over the entire surface of a body equal the total force exerted by the fluid on that body. Remembering that fact when we get back to lift can avoid lots of confusion.

A stress is a force; a strain is deformation of a material in response to that force. In a fluid, a constant force produces a constant rate of strain $[\text{L T}^{-1}]$. In a solid it produces a constant strain $[\text{L}^{-1}]$.

$$\text{Momentum or kinetic energy} = (mv^2)/2; (\text{mass} \times \text{velocity}^2)/2 [\text{M L}^2 \text{T}^{-2}]. \quad (3)$$

First, let me comment on the dimensions. You know that energy and mass are interconvertible, even if you don't want to try this trick at home: $E = mc^2$. So if you can remember the formula, you can give the dimensions for energy: A mass times a velocity squared (c being the speed of light) is $[\text{M} (\text{L/T})^2] = [\text{M L}^2 \text{T}^{-2}]$. But why does the equation for momentum or kinetic energy of water or mass of any other sort have a denominator of two? Start a body from rest; kinetic energy = 0. Now apply a force over some time, t , and you will accelerate the body to some velocity v . By definition, the object's acceleration, $a = (\text{change in speed}/\text{time over which speed was changing}) = v/t$. So you already know that $F = mv/t$. The average speed of the object over t is simply $v/2$. What distance did it go over t if its speed is $v/2$? Simply multiply the speed times the time to get the distance (check dimensions): $vt/2$. Work and energy have the same dimensions. Now multiply the force times the distance:

$$\frac{mv}{t} \times \frac{vt}{2} = \frac{mv^2}{2}. \quad (4)$$

If you divide momentum by volume you get momentum per unit of volume $[(\text{M L}^2 \text{T}^{-2})/\text{L}^3 = \text{M L}^{-1} \text{T}^{-2}]$ for a fluid moving with the specified velocity. In terms of variables that you see in the book (Eq. 4.2, p. 53), momentum/volume = $\rho U^2/2$. Notice that the dimensions are the same as those of pressure, so you get a sense that a pressure as a kind of potential energy in a volume is convertible into a velocity of that volume of fluid as kinetic energy. Bernoulli's law is an expression of this interconvertibility; for the usual, frictionless (inviscid) version of the law, the interconversion is perfectly efficient.

Pressure at the front of an object (that is moving relative to the water) is high because momentum has been extracted from the fluid and converted to pressure. If you always worry about the *relative* flow velocity, you will not confuse the front (upstream or windward) and back (downstream or leeward) faces of a body. Pressure drops at the sides because kinetic energy is gained at the expense of pressure (so the pressure does not continue to rise at the front, but instead drives the flow around). Pressure in a real fluid fails to return to its frontal maximum at the rear because friction extracts energy and because, as you saw most clearly for very low Reynolds numbers, a departing fluid that has viscosity pulls on the object (exerts tensile stresses on the object in the form of low pressure). Above very low Re , the wake is in general less organized than the oncoming flow (having higher entropy, with turbulence being dissipated to laminar shear and in turn to heat).

In a real fluid and the real plumbing of your home or irrigation system assume that the pipes all are horizontal, except for the inflow from a head tank and that all pipe diameters are the same and constant. Set up an initial flow rate under a constant head of pressure. Now keep that (driving) head of pressure constant and add more horizontal pipe. The flow rate out the end of the pipe (and everywhere else in your pipe) will decrease with the added length of pipe because you are adding frictional losses to the system but not adding any driving force. The difference in pressure between the inlet and outlet ends of your pipe will therefore increase. Let's say that as a member of the volunteer fire department you find this reduced rate of flow unacceptable. To return the flow rate to its original, you will have to raise the head tank. That is, if you keep the head of pressure the same, in a longer pipe you will get a slower flow rate. Conversely, if you want to keep the flow rate the same, you will need a higher head of pressure. Exercise your understanding by putting these ideas together with the diagram (Fig. 4.2) on p. 53 of your text. Fig. 4.2 does not suggest any frictional losses.